19 at 0.010

width [. . . . .]

LRAM = 898
RRAM = 1040

avg. = 969 · 0.001 = 0.969 miles

0.4845

at 0.006

LRAM sum 0 − 0.005 = 338
RRAM sum 1 − 0.006 = 504

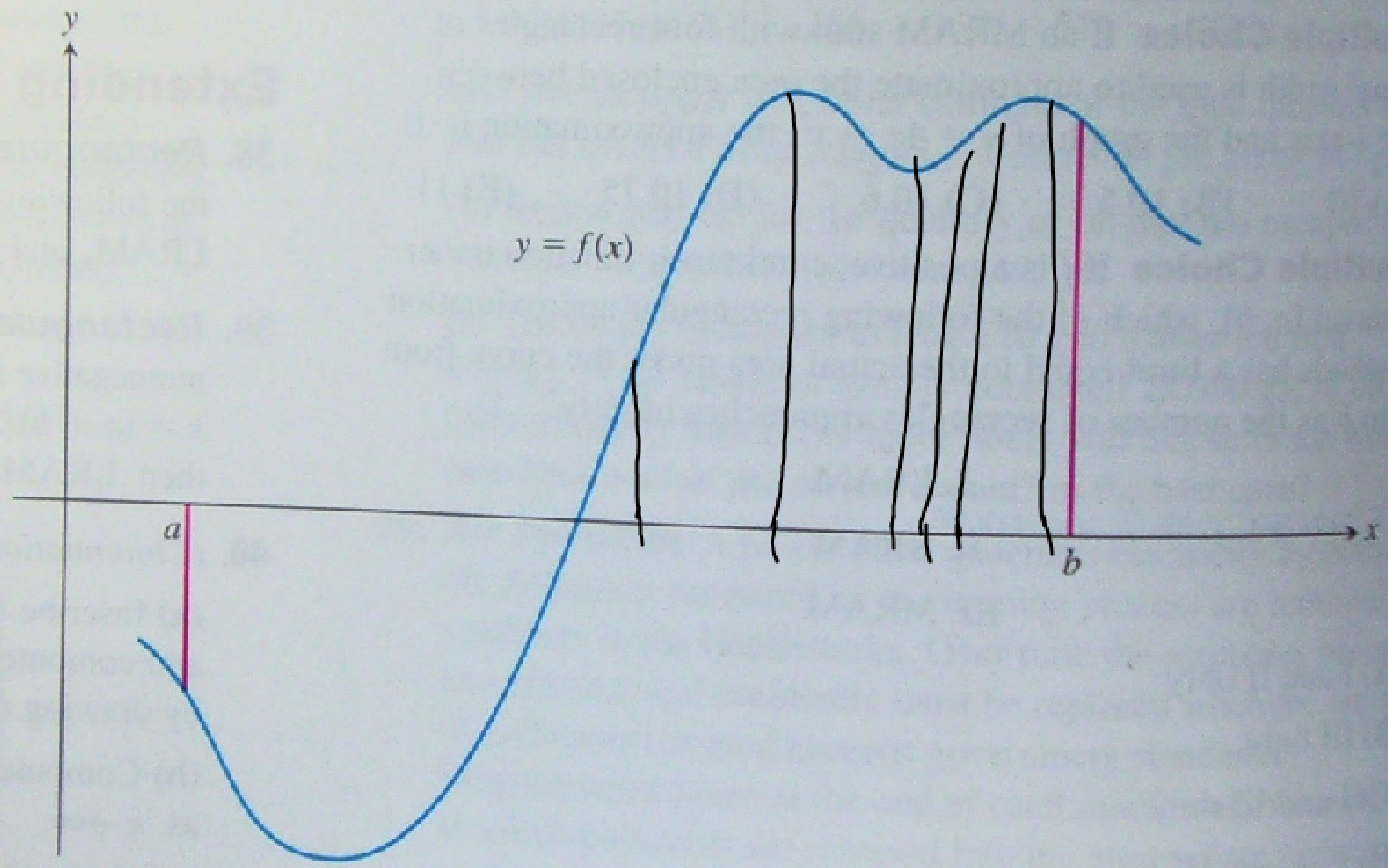
avg = 421 · 0.001 = 0.421

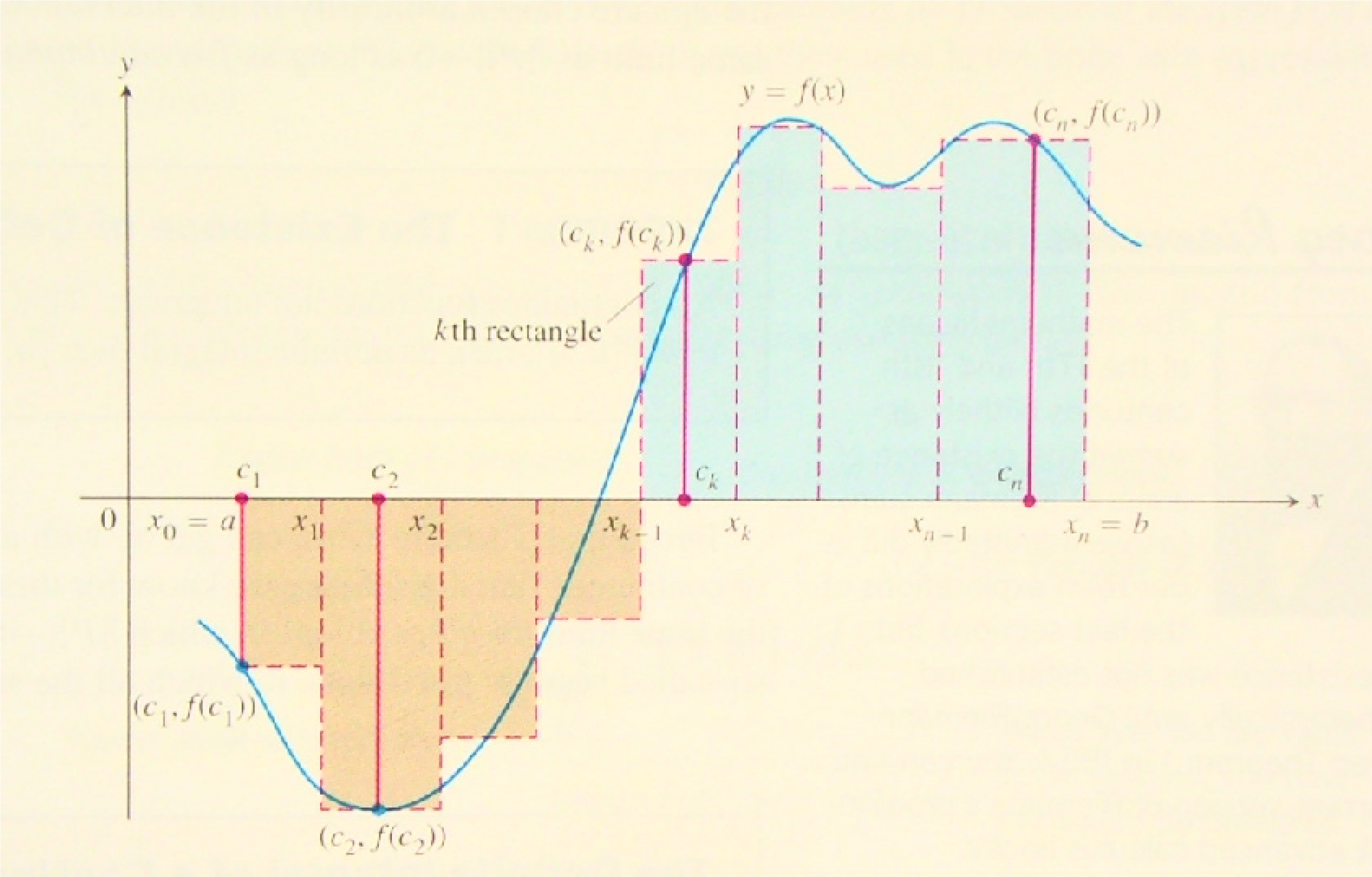
$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \cdots + a_{n-1} + a_n.$$

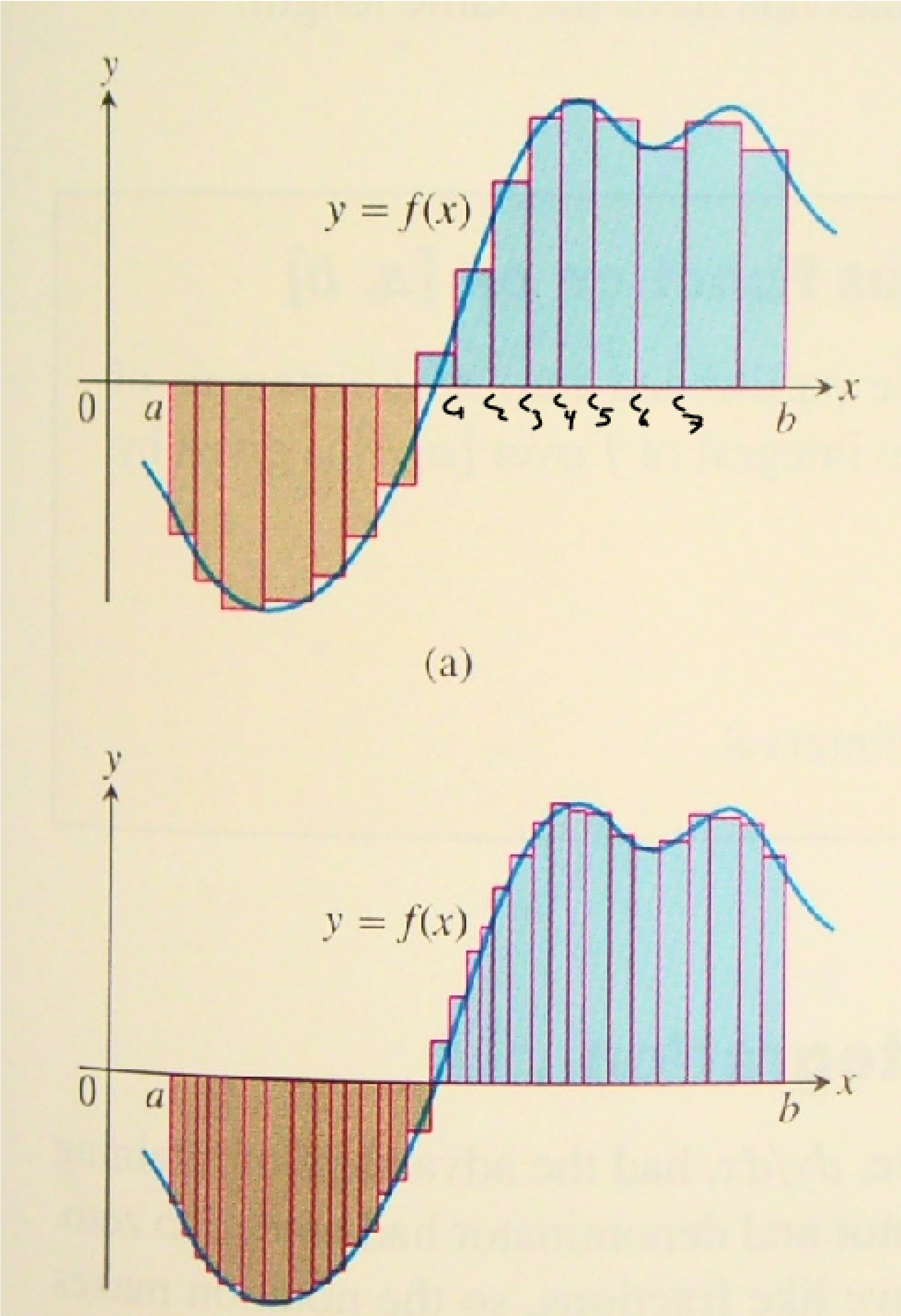
$$\sum_{i=1}^{5} i^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$$

$$\sum_{i=1}^{5} 2i = 2, 4, 6, 8, 10$$

We begin with an arbitrary continuous function $f(x)$ defined on a closed interval [a,
Like the function graphed in Figure 5.12, it may have negative values as well as posi
values.




y
y = f(x)
(c_n, f(c_n))
(c_k, f(c_k))
kth rectangle
c_1
c_2
c_k
c_n
x
0
x_0 = a
x_1
x_2
x_{k-1}
x_k
x_{n-1}
x_n = b
(c_1, f(c_1))
(c_2, f(c_2))



$$S_n = \sum_{k=1}^{n} \underbrace{f(c_k)}_{\text{height of rect.}} \cdot \underbrace{\Delta x_k}_{\text{width}}.$$

Rieman Sum

DEFINITION The Definite Integral as a Limit of Riemann Sums

Let f be a function defined on a closed interval $[a, b]$. For any partition P of $[a, b]$, let the numbers c_k be chosen arbitrarily in the subintervals $[x_{k-1}, x_k]$.

If there exists a number I such that

$\|P\| \to$ width of widest rect.

$$\lim_{\|P\| \to 0} \sum_{k=1}^{n} f(c_k)\,\Delta x_k = I$$

no matter how P and the c_k's are chosen, then f is **integrable** on $[a, b]$ and I is the **definite integral** of f over $[a, b]$.

THEOREM 1 The Existence of Definite Integrals

All continuous functions are integrable. That is, if a function f is continuous on an interval $[a, b]$, then its definite integral over $[a, b]$ exists.

The Definite Integral of a Continuous Function on [a, b]

Let f be continuous on $[a, b]$, and let $[a, b]$ be partitioned into n subintervals of equal length $\Delta x = (b - a)/n$. Then the definite integral of f over $[a, b]$ is given by

$$\lim_{n \to \infty} \sum_{k=1}^{n} f(c_k)\,\Delta x,$$

where each c_k is chosen arbitrarily in the k^{th} subinterval.

The symbol

$$\int_a^b f(x)\,dx$$

is read as "the integral from a to b of f of x dee x," or sometimes as "the integral from a to b of f of x with respect to x." The component parts also have names:

Upper limit of integration

The function is the **integrand.**

Integral sign

x is the **variable of integration.**

$$\underbrace{\int_a^b f(x)\,dx}_{\textbf{Integral}\text{ of } f \text{ from } a \text{ to } b}$$

Lower limit of integration

When you find the value of the integral, you have **evaluated the integral.**

EXAMPLE 1 Using the Notation

The interval $[-1, 3]$ is partitioned into n subintervals of equal length $\Delta x = 4/n$. Let m_k denote the midpoint of the k^{th} subinterval. Express the limit

$$\lim_{n \to \infty} \sum_{k=1}^{n} (3(m_k)^2 - 2m_k + 5)\, \Delta x$$

∂x

as an integral.

continued

Slope $\frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$ when $\to \infty$ $\frac{\partial y}{\partial x}$

DEFINITION **Area Under a Curve (as a Definite Integral)**

If $y = f(x)$ is nonnegative and integrable over a closed interval $[a, b]$, then the **area under the curve $y = f(x)$ from a to b** is the integral of f from a to b,

$$A = \int_a^b f(x)\,dx.$$

$$\text{Area} = -\int_a^b f(x)\,dx \quad \text{when} \quad f(x) \le 0.$$

$$\int_a^b f(x)\,dx = (\text{area above the } x\text{-axis}) - (\text{area below the } x\text{-axis}).$$

positive

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THEOREM 2 The Integral of a Constant

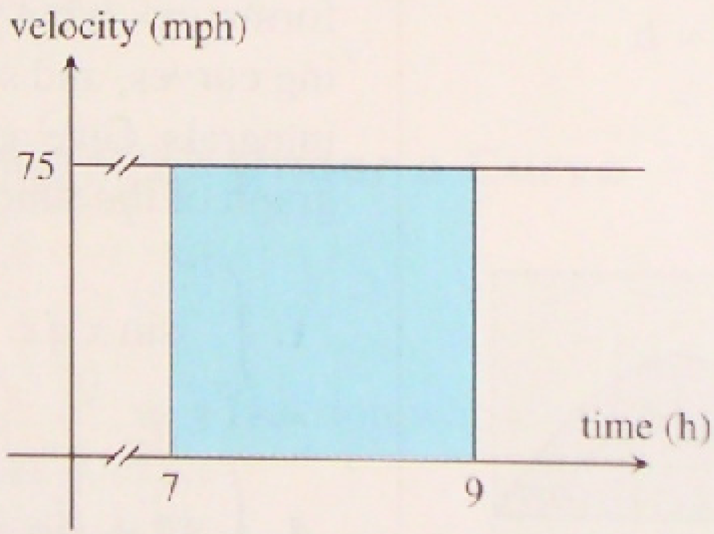
If $f(x) = c$, where c is a constant, on the interval $[a, b]$, then

$$\int_a^b f(x)\,dx = \int_a^b c\,dx = \overset{\substack{\text{height}\\ \Downarrow}}{c}\underbrace{(b - a)}_{\text{width}}.$$

EXAMPLE 3 Revisiting the Train Problem

A train moves along a track at a steady 75 miles per hour from 7:00 A.M. to 9:00 A.M. Express its total distance traveled as an integral. Evaluate the integral using Theorem 2.

SOLUTION (See Figure 5.22.)



MATH

EXAMPLE 4 Using NINT

Evaluate the following integrals numerically.

(a) $\int_{-1}^{2} x \sin x \, dx$ **(b)** $\int_{0}^{1} \frac{4}{1 + x^2} \, dx$ **(c)** $\int_{0}^{5} e^{-x^2} \, dx$

SOLUTION

(f(x), x, a, b)

(a) NINT $(x \sin x, x, -1, 2) \approx 2.04$

(b) NINT $(4/(1 + x^2), x, 0, 1) \approx 3.14$

(c) NINT $(e^{-x^2}, x, 0, 5) \approx 0.89$

No

Sect. 5.2

#1-28 (pick 4 each section), 33-36